## Functions and Limits Review

In order to do well in this course and before we can understand limits, we must know our basic functions. Here is a quick visual review of graphs of some functions that students sometimes forget:









On the exams, you are allowed to use what you see in these graphs. For example, by looking at the graphs you immediately know all of the following:

| $\lim _{x \rightarrow-\infty} e^{x}=0$ | $\lim _{x \rightarrow \infty} e^{x}=\infty$ | $\lim _{x \rightarrow 0^{+}} \ln (x)=-\infty$ | $\lim _{x \rightarrow \infty} \ln (x)=\infty$ |
| :--- | :--- | :--- | :--- |
| $\lim _{x \rightarrow-\infty} \frac{1}{x}=0$ | $\lim _{x \rightarrow 0^{-}} \frac{1}{x}=-\infty$ | $\lim _{x \rightarrow 0^{+}} \frac{1}{x}=\infty$ | $\lim _{x \rightarrow \infty} \frac{1}{x}=0$ |
| $\lim _{x \rightarrow \frac{\pi}{2}-} \tan (x)=\infty$ | $\lim _{x \rightarrow-\frac{\pi}{2}} \tan (x)=-\infty$ | $\lim _{x \rightarrow \infty} \tan ^{-1}(x)=\frac{\pi}{2}$ | $\lim _{x \rightarrow-\infty} \tan ^{-1}(x)=-\frac{\pi}{2}$ |

## One Special Note:

We will make use of the particular fact $\lim _{x \rightarrow 0} \frac{\sin (x)}{x}=1$ (if $x$ is in radians!).
A proof of this fact is posted on my course website and is in the book. The variable $x$ is not important, what this says is that $\lim _{B L A H \rightarrow 0} \frac{\sin (B L A H)}{B L A H}=1$. So for example: $\lim _{x \rightarrow 0} \frac{\sin (10 x)}{10 x}=1$ and $\lim _{x \rightarrow 0} \frac{\sin (31 x)}{31 x}=1$.

Now if the denominator does not match the numerator, then we can do a bit of rearranging of fractions to make them match.
For example: $\lim _{x \rightarrow 0} \frac{\sin (5 x)}{x}=\lim _{x \rightarrow 0} \frac{\sin (5 x)}{x} \frac{5}{5}=\lim _{x \rightarrow 0} 5 \frac{\sin (5 x)}{5 x}=5 \cdot 1=5$.

## Practice, practice, practice!:

Using the facts above along with the limit strategies discussed in class and summarized in my other review sheets, go practice your limit methods. There is a compilation of old final problems posted online, check them out. Also look at old midterm exams. Besides the departmental old exam archive, I also maintain my own archive of many old exams by me and other instructors I know, check out many, many, many old exams. You need to expose yourself to lots of different problems!

